

# Economic Uncertainties in Chilled Water System Design

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## ABSTRACT

*The analysis described here examines how uncertainties in engineering and economic assumptions made during chilled water system design translate to uncertainty in commonly used design decision metrics. The metric used is the benefit-cost ratio based on discounted cash flow. This analysis was performed as part of a project that is developing engineering tools for use in selecting energy-efficient chilled water system components, controls, and operating strategies. These tools include cooling thermal load prediction capabilities and performance data and models for chillers and cooling towers. The purpose of this study is to estimate accuracy requirements for the load and performance data that will be provided as part of the chilled water system tools. The logic is that there is inherent uncertainty in the decision metric due to uncertainty in inputs other than load and equipment performance, and, consequently, there is a limit below which further improvements in the accuracy of the load and equipment performance do not appreciably improve the quality of information available to the decision maker.*

## INTRODUCTION

Uncertainty in decision metrics, such as the benefit-cost ratio or simple payback, is a measure of risk that the designer and owner face in selecting a chilled water system design. Of course, these economic metrics are not the sole basis for decisions in most cases. The decision whether to install a particular technology is part of a broader set of economic and non-economic considerations, the parameters of which are unique to the project and to the ownership environment. Though the decision regarding a particular technology is not separable from a host of other decisions, the value assigned to the

metrics used by the decision maker, the confidence that he or she has in that value, and the attitude toward risk, play roles in defining where the technology decision is located in the pecking order of decisions. For this reason it is important that attempts to improve the accuracy of cooling load and equipment performance data used in chilled water system design be accompanied by efforts to understand how those improvements impact the risk faced by the owner and designer.

That uncertainty is a significant issue is borne out by a study of decision making in the industrial sector (Peters et al. 1996). Key barriers identified in this sector are expected to be important for the commercial sector, too. Among the barriers to energy investment decision, risk was identified as key. The stage of the business cycle that the company was in at the time of the decision was also identified as an important consideration for the decision maker.

The methods of analysis used in this study are consistent with those recommended by the American Society for Testing and Materials (ASTM) in a series of standards (ASTM 1997, 1993a, 1993b). These standards identify commonly used methods of economic analysis and describe procedures for using benefit-cost analysis. ASTM standards also exist for other methods of economic analysis. Risk analysis is the topic of a separate ASTM standard (ASTM 1993c). This standard describes methods for dealing with uncertainty in the independent variables used in calculating common economic metrics and is based on work at the National Institute of Standards and Technology (NIST) (Marshall 1988).

The techniques used in the uncertainty analyses described here are closely related to those used to study measurement uncertainty and its effects on experimental results. These procedures are described in *ASHRAE Guide-line 2-1986 (RA 90)* (ASHRAE 1990) and in *ANSI/ASME*

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PTC 19.1-1985 (ASME 1985). Considerably more comprehensive treatments of these methods are also available (see, for example, Dieck [1997] and Coleman and Steele [1989]).

Uncertainty analysis techniques have been regularly applied in some areas. In energy-related fields, they have been used by utility planners where there is substantial uncertainty in future energy requirements that can have appreciable impact on capacity requirements (see, for example, Violette and Olsson [1994]). At a somewhat more detailed level, the techniques have been used in demand-side management program design to inform resource allocation decisions (see, for example, Pigg et al. 1995). Sensitivity analyses have been used to examine the uncertainty in estimates of energy conservation potential (Norris 1996).

Uncertainty analysis methods have been used to estimate the uncertainty in building simulation estimates (Stern et al. 1994). In this study, simulation inputs were varied in order to estimate influential coefficients (Spitler et al. 1989) that describe the sensitivity of the energy requirement estimates from the simulation to the uncertain input parameters. The uncertainties were then propagated to the savings estimates. In a case study application of the method, Stern estimated an uncertainty of about 30% for the savings estimated by simulation for a ceiling insulation retrofit in residential buildings. This is not a general conclusion; however, it highlights the fact that these uncertainties can be substantial, and they will directly affect the attractiveness of energy technology.

## APPROACH

The approach consists of developing an expression for the benefit-cost ratio as a function of the independent engineering and economic variables. The sensitivity of the metric to individual input parameters is determined from this expression. Uncertainties are estimated for each parameter and propagated to the benefit-cost ratio using those sensitivities. This provides in an estimate the range over which the decision metrics could vary.

## Benefit-Cost Ratio

In a situation where one is attempting to select between a baseline chilled water system and a more efficient option, one metric that can be used by the decision maker is the benefit-cost ratio based on discounted cash flows. This can be written as follows:

$$B/C = \frac{\sum_{i=1}^{8760} [L_i \cdot (f_{B,L,i} - f_{E,L,i}) \cdot P_i] \cdot D_{N,r}}{\frac{C_F}{(1+r)} + C_{Ann} \cdot D_{N,r}} \quad (1)$$

where

B/C = benefit-cost ratio,

$i$  = hour of year ( $1^2 i^2 8760$ ),

$L_i$  = thermal load on chilled water system at hour  $i$ ,

$f_{B,L,i}$  = efficiency of the base case system in conversion of kW to Btu,

$f_{E,L,i}$  = efficiency of the high-efficiency system in conversion of kW to Btu,

$P_i$  = electricity price at hour  $i$ ,

$C_F$  = total incremental first cost for the technology,

$C_{Ann}$  = total incremental annual operating cost (excluding energy costs) for the technology,

$r$  = discount rate.

$D_{N,r}$  is the discount factor:

$$D_{N,r} = \sum_{n=1}^N \frac{1}{(1-r)^n} \quad (2)$$

where  $n$  is the year and  $N$  is the period of analysis.

This formulation of the benefit-cost ratio adopts a simple accounting perspective. The benefits considered are limited to energy cost savings directly attributable to selection, configuration, and operation of chilled water equipment that maximizes system efficiency. Similarly, costs considered are limited to the additional (1) first cost for design, installation, and commissioning of the chilled water system and (2) non-energy operating costs for the efficient chilled water system (e.g., additional maintenance requirements). It is emphasized that the costs and benefits are incremental quantities referenced to the energy costs, first cost, and operating cost requirements for an alternative to the base case technology.

The subscripts  $L$  and  $i$  on the conversion efficiency allow for dependence on instantaneous load and climatic conditions; by not including the subscript  $n$ , we are assuming that there is no change in performance of equipment over time (i.e., degradation). The formulation does not explicitly account for demand charges, although they could be approximated by adjusting the electricity price parameter. For simplicity, we have assumed that load and price are cyclic on an annual basis, so the subscript  $n$  has been suppressed for these parameters—this assumes that there is no “evolution” of the cooling load as the technological environment changes over time (i.e., no load reduction due to future installation of energy efficiency measures and no alteration in the energy use intensity of functional and process equipment).

We assume that the incremental non-energy operating costs for the technology are the same each year. For simplicity, incremental costs that recur with a period greater than one year are neglected (e.g., major maintenance procedures). In practice, these costs may be somewhat higher for a more sophisticated design. However, we expect that most of the difference observed in practice would be due, not to differences in maintenance recommendations by manufacturers, but to differences in the way those requirements are interpreted and adhered to by the maintenance staff. There are probably exceptions, and one could approximate their effect by prorating those costs over the recurring cost period.

By including in the denominator of the benefit-cost ratio all incremental first costs and operating costs, one is assuming that the decision maker requires a return on all of these investments at least as large as the discount rate used in the calculation. There are alternative formulations where the discounted annual operating costs are included instead as subtractive terms in the numerator, leaving first costs as the only term in the denominator. This formulation is used in a situation where the decision maker requires a return only on the initial capital expenditure. In still other cases, where some of the first costs are financed, discounted financing costs are also subtracted from the benefits in the numerator and only the nonfinanced portion of the first costs are included in the denominator as funds on which a return is expected. Independent of which of these formulations is used, a benefit-cost ratio less than one indicates that the technology is economically unattractive. The larger the benefit-cost ratio, the more attractive the technology.

This formulation of the benefit-cost ratio is consistent with the convention that costs are accounted for at the end of each year. The formulation assumes that all of the initial costs occur at the end of the first year. This is consistent with the reasonable assumption that the benefit-cost ratio is calculated during the decision-making process, prior to incurring most of the first costs. The formulation also assumes that there is a full year of benefits during the first year. The scale of the installation, specific details of the project, and financing terms would determine the appropriateness of these assumptions. Taken together, they imply that the benefit-cost ratio calculated from the above expression would be somewhat optimistic.

Inflation and escalation have not been included explicitly in this formulation. However, since we are dealing with present values of future costs and savings, inflation effects would cancel and so are accounted for implicitly. This is not true for escalation unless all of the economic parameters entering the cost and benefit calculation serendipitously escalate at the same rate. This implies that the discount rate used in the calculations should be the real rate rather than the nominal rate. There also is no explicit accounting for taxes and depreciation, so it is important to use a before-tax discount rate.

The above expression for the benefit-cost ratio holds, independent of what tools the designer uses. In the final analysis, there is one and only one correct value for the benefit-cost ratio—and if we had kept all of the data (and were still interested), we could, in principle, calculate its actual value at the end of the analysis period. However, this *ex post facto* value of the benefit-cost ratio is irrelevant; the decision of whether to use the more efficient equipment option must be made based on the benefit-cost ratio defined by design estimates for the input parameters.

### Uncertainty Analysis

In this study, we are examining how uncertainties in the input parameters propagate to the value calculated for the benefit-cost ratio. That is, we are examining how improved

tools affect risk. We are not estimating the extent to which the use of these tools will alter the benefit-cost ratio by leading to a better design; that is clearly a highly application-specific issue that is beyond the scope of the present study.

In calculating the benefit-cost ratio, uncertainty will be associated with all of the parameters that enter the calculation. Whether one uses standard practice tools, methods, and calculations, or more advanced tools, there will be some degree of uncertainty in each of the input parameters, denoted by  $\Delta L_{\dot{p}}$ ,  $\Delta f_{B, L, i}$ ,  $\Delta f_{E, L, \dot{p}}$ ,  $\Delta P_{\dot{p}}$ ,  $\Delta C_F$ ,  $\Delta C_{Ann}$ ,  $\Delta N$ , and  $\Delta r$ . These component uncertainties are propagated to the benefit-cost ratio using standard Taylor Series expansion techniques. As is common practice, we retain only the first-order terms in the expansion:

$$B/C = (B/C)|_A + \sum_{k=1}^m \left[ \Delta\chi_k \cdot \left. \frac{\partial(B/C)}{\partial\chi_k} \right|_{\chi_A} \right] \quad (3)$$

where  $\chi_k$  are the  $m$  independent variables in the benefit-cost calculation, and where the benefit-cost and partial derivative terms on the right are evaluated at the point  $\chi_A = (\chi_{1, A}, \chi_{2, A}, \dots, \chi_{n, A})$  and  $\Delta\chi_k = \chi_k - \chi_{k, A}$ . Note that although the notation is not explicit in this regard, the expansion describes the benefit-cost ratio in the immediate vicinity of the ratio at the particular set of inputs,  $\chi_A$ , that is, for small values of  $\Delta\chi_k$ .

Rearranging the expansion,

$$\Delta(B/C) = \sum_{k=1}^m \left[ \Delta\chi_{ki} \cdot \left. \frac{\partial(B/C)}{\partial\chi_k} \right|_{\chi_A} \right] \quad (4)$$

This expression describes the deviation of the benefit-cost ratio from its value at point  $\chi_A$  due to small deviations,  $\Delta\chi_k$ , in an individual independent variable from its nominal values at  $\chi_A$ . For a particular set of deviations from  $\chi_A$ , the individual  $\Delta\chi_k$  can take positive or negative values. In a specific situation, we do not have complete information—we do not know the direction of the deviations. The standard procedure is to square both sides of the expression to obtain, after simplification,

$$\begin{aligned} [\Delta(B/C)]^2 = & \sum_i \left[ \Delta\chi_i \cdot \left. \frac{\partial(B/C)}{\partial\chi_i} \right] \right]^2 \\ & + 2 \cdot \sum_{j, k \neq j} \frac{\partial(B/C)}{\partial\chi_j} \cdot \left. \frac{\partial(B/C)}{\partial\chi_k} \right] \cdot R_{j, k} \cdot \Delta\chi_j \cdot \Delta\chi_k \end{aligned} \quad (5)$$

where  $R_{j, k}$  is the correlation coefficient of  $\Delta\chi_j$  on  $\Delta\chi_k$ . Note that although the notation has been altered for clarity, the partial derivative terms on the right side of the equation are evaluated at point  $\chi_A$  as before.

In this analysis, the uncertainties in the individual parameters are assumed to be independent, so the second term vanishes. For uncertainty in a single parameter,  $\chi_j$ , the resulting uncertainty in the benefit-cost ratio will be estimated from

$$\Delta(B/C) = \pm \Delta\chi_k \cdot \left. \frac{\partial(B/C)}{\partial\chi_k} \right] \quad (6)$$

To estimate the total uncertainty in the benefit-cost ratio due to all sources when component uncertainties are assumed random, the component uncertainties will be added in quadrature:

$$\Delta(B/C) = \sqrt{\sum_{k=1}^m \left[ \Delta\chi_k \cdot \frac{\partial(B/C)}{\partial\chi_k} \right]^2} \quad (7)$$

In many cases, the uncertainty in the individual parameters is expected to be systematic rather than random. Where both systematic and random uncertainties are present, there are accepted procedures for combining the individual uncertainties into a single uncertainty.

The uncertainties in some input parameters will be situation-specific. For example, the load uncertainty in a retrofit situation may be smaller than in new construction because one can monitor the load in an existing building. Monitoring would decrease uncertainties (and therefore risk) at some increase in the first cost for the retrofit. The attractiveness of this trade-off is undoubtedly situation-specific. As another example, the equipment performance uncertainty in a situation where a chiller is being replaced should be smaller than when the chilled water system as a whole is being replaced. In this analysis, we consider only a new construction scenario. We note that uncertainties in discount rate, analysis period, and electricity price in general will be independent of the situation.

## BENEFIT-COST UNCERTAINTY ANALYSIS

In what follows, we will go into some detail in considering the uncertainty in first cost and will provide summaries of the uncertainty calculations for the other independent variables.

### First Cost Uncertainty

The first cost includes all costs incurred to achieve a working system. This would include design, purchase, installation, and commissioning costs. In a retrofit situation, where monitoring is performed to characterize loads, monitoring and analysis costs would also be included in the first cost category. The incremental first cost is the total increase in these costs in going from the base case chilled water system to the efficient option.

The sensitivity of the benefit-cost ratio to “small” uncertainties in first cost is

$$\frac{\partial(B/C)}{\partial C_F} = -(B/C) \cdot \frac{1}{C_F + (1+r) \cdot C_{Ann} \cdot D_{N,r}} \quad (8)$$

After simplification, the fractional uncertainty in benefit-cost ratio due to an uncertainty in first cost can be estimated as

$$\frac{\Delta(B/C)}{(B/C)} = \frac{-\Delta C_F}{C_F + C_{Ann} \cdot (1 + D_{N-1,r})} \quad (9)$$

The incremental first cost uncertainty,  $\Delta C_F$ , can be expressed in terms of the uncertainties in the first cost for the

base case and efficient equipment options,  $\Delta C_{B,F}$  and  $\Delta C_{E,F}$ , respectively,

$$\Delta C_F = \sqrt{(\Delta C_{B,F})^2 + (\Delta C_{E,F})^2} \quad (10)$$

Assuming the same cost estimating procedures are used for the baseline and efficient equipment options, one expects

$$\frac{\Delta C_{B,F}}{C_{B,F}} \approx \frac{\Delta C_{E,F}}{C_{E,F}} \quad (11)$$

Thus,

$$\Delta C_F \approx \Delta C_{E,F} \cdot \sqrt{1 + \left( \frac{C_{F,B}}{C_{E,B}} \right)^2} \quad (12)$$

For typical situations, the first cost penalty for energy-efficient equipment options is not more than perhaps 25%:

$$0.75 \leq \frac{C_{B,F}}{C_{E,F}} \leq 1 \quad (13)$$

so  $\Delta C_F$  will be in the range from  $1.25 \cdot \Delta C_{E,F}$  to  $1.41 \cdot \Delta C_{E,F}$ . For convenience, we take

$$\Delta C_F \approx 1.35 \cdot \Delta C_{E,F} \quad (14)$$

We define dimensionless parameters  $n$  and  $m$ :

$$\Delta C_{E,F} = n \cdot C_{E,F} \quad (15)$$

$$C_{Ann} = m \cdot C_{E,F} \quad (16)$$

We assume that the same dimensionless parameter  $m$  describes the relationship between annual operating cost for the base case system and the first cost for that system. Then:

$$\frac{\Delta(B/C)}{(C/C)} = \frac{-1.35 \cdot n}{1 + m \cdot (1 + D_{N-1,r})} \cdot \frac{C_{E,F}}{C_{B,F} - C_{E,F}} \quad (17)$$

This expression has been evaluated for a realistic range of values for the dimensionless parameters<sup>1,2</sup> in the equation:

$$2\% \leq n \leq 10\%$$

$$3\% \leq m \leq 9\%$$

$$4 \leq D_{N,r} \leq 12$$

$$0.65 \leq (C_F^B / C_F^E) \leq 0.95 \quad (18)$$

1. For discount rates and analysis periods typical of these types of analysis (e.g.,  $5\% \leq r \leq 20\%$ ,  $5 \leq N \leq 20$  years), the discount factor is  $3 \leq D_{N,r} \leq 12$ .  $D_{N,r}$  takes on large values when the period of analysis is long and the discount rate is small, and  $D_{N,r}$  is small for short periods of analysis and high discount rates.

2. The choice of the range for the parameter  $n$  is based on estimates of contingency funds typically included in construction budgets. R. S. Means (1998) indicates contingency allowances of 15% at conceptual design, 10% at schematics, 7% at the preliminary working drawing stage, and 2% at final working drawings.

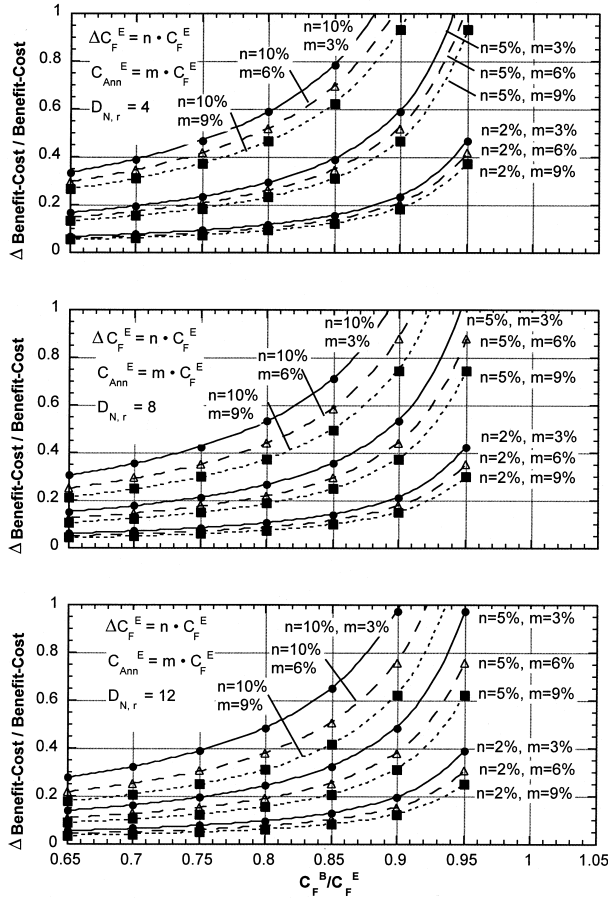


Figure 1 Results of the numerical evaluations.

The results of the numerical evaluations are presented in Figure 1. Note that the general shape of these curves, with the rapidly increasing slope as  $(C_F^B / C_F^E)$  approaches 1, is due to the fact that we are examining the uncertainty in the difference between two large numbers. The uncertainty in the difference between these numbers due to uncertainties in the numbers themselves becomes large as the difference between the numbers becomes small.

For intermediate periods of analysis (i.e., 10 to 15 years) and modest discount rates (6% to 10%), the center graph ( $D_{N,r} = 8$ ) is appropriate. The choice of the remaining parameters in Figure 1 will be dependent on the chilled water system installation scenario (i.e., new vs. retrofit). For new construction, we estimate first cost uncertainty for chilled water system installation at perhaps  $\pm 5\%$  at the stage of design where alternative plants are being considered. We estimate the first cost premium for the efficient chilled water system at perhaps 20%. This translates to a benefit-cost uncertainty of about  $\pm 25\%$  due to uncertainty in first cost.

### Annual Operating Cost Uncertainty

The annual operating cost includes all costs other than those for electrical energy for operation. This would include labor and disposable materials associated with maintenance of

the equipment. Note again that the analysis considers only the incremental operating cost for the efficient technology option relative to the baseline option.

As with first cost, uncertainty in incremental annual non-energy operating costs is expressed in terms of the uncertainty in the absolute operating costs for the base case and efficient systems. As before, we assume that for a particular cost-estimating procedure, the fractional uncertainty in the annual non-energy operating costs for the base case and efficient systems will be essentially the same. Furthermore, we expect,<sup>3</sup>

$$C_{B,Ann} \approx C_{E,Ann} \quad (19)$$

and so,

$$\Delta C_{Ann} \approx 1.4 \cdot \Delta C_{E,Ann} \quad (20)$$

The benefit-cost uncertainty can then be written:

$$\frac{\Delta(B/C)}{(B/C)} = \frac{-1.4 \cdot n \cdot m \cdot D_{N-1,r}}{1 + m \cdot (1 + D_{N-1,r})} \cdot \frac{C_{E,F}}{C_{B,F} - C_{E,F}} \quad (21)$$

As before, the dimensionless parameter  $n$  is the fractional uncertainty in  $C_{E,Ann}$  and  $m$  is the fraction of  $C_{E,F}$  that is required for annual non-energy operating costs. We assume the same value of  $m$  describes the annual non-energy operating cost for the base case and efficient system options.

This expression has been evaluated for a realistic range of values for the dimensionless parameters:

$$2\% \leq n \leq 10\%$$

$$3\% \leq m \leq 9\%$$

$$4 \leq D_{N,r} \leq 12^4$$

$$0.65 \leq (C_F^B / C_F^E) \leq 0.95 \quad (22)$$

The results of these numerical evaluations are presented in Figure 2.

Again, considering intermediate analysis periods (i.e., 10 to 15 years) and relatively low discount rates (6% to 10%), the center graph ( $D_{N,r} = 8$ ) is appropriate. The choice of the remaining parameters in Figure 2 is expected to be dependent on the chilled water system installation scenario. We estimate operating cost uncertainty for chilled water systems in new construction at perhaps  $\pm 5\%$ . As before, assuming a first cost

3. Note that this does not imply that baseline systems will actually receive the same amount of maintenance attention as the efficient system in practice. Instead, it is a statement that the choice of an efficient chilled water system does not, by itself, increase maintenance requirements in most cases. Note also that this is not true for some technologies (e.g., engine-driven chillers).

4. For discount rates and analysis periods typical of these types of analysis (e.g.,  $5\% \leq r \leq 20\%$ ,  $5 \leq N \leq 20$  years), the discount factor is  $3 \leq D_{N,r} \leq D_{N,r} \leq 12$ .  $D_{N,r}$  takes on large values when the analysis period is long and the discount rate is small, and  $D_{N,r}$  is small for short analysis periods and high discount rates.

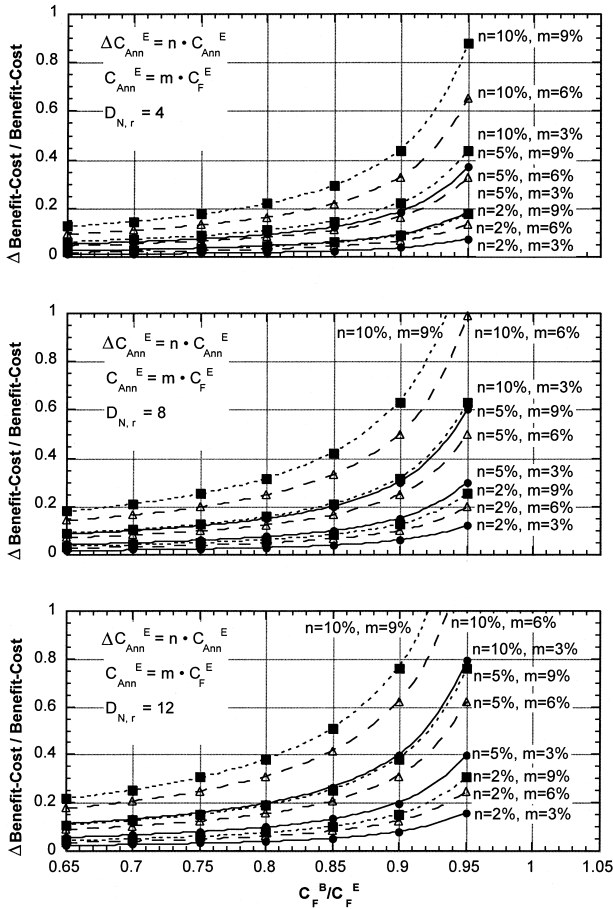


Figure 2 Results of the numerical evaluations.

premium for the efficient chilled water system of 20%, the uncertainty in the benefit-cost ratio due to uncertainty in non-energy annual operating costs will be about  $\pm 15\%$ .

### Electric Price Uncertainty

We assume that the hourly prices are discrete, with possibly different values for each hour. Although we cannot differentiate price with time (because it is not a continuous function of time) we can differentiate with respect to a price parameter,  $p(t)$ , where

$$P_i = p(t)|_{t=i} \quad (23)$$

The differential with respect to  $p$  is well defined because the price at any time is assumed to be able to take on a continuous range of values. The uncertainty in the benefit-cost ratio due to an uncertainty in price,  $\Delta p$ , is given by

$$\Delta(B/C) = \frac{8760 \sum_{i=1} [L_i \cdot (f_{B,L,i} - f_{E,L,i}) \cdot \Delta p \cdot D_{N,r}]}{\frac{C_F}{1+r} + C_{Ann} \cdot D_{N,r}} \quad (24)$$

The benefit-cost uncertainty depends explicitly on the nature of the price uncertainty. In a case where  $\Delta p = \delta$  (i.e., the price uncertainty is an offset from the expected price), the benefit-cost uncertainty is

$$\Delta(B/C) = (B/C)|_{P_i = \$\delta/kWh} \quad (25)$$

In a case where  $\Delta p = \varepsilon \cdot p$  (i.e., the price uncertainty is proportional to the expected price), the benefit-cost uncertainty is

$$\Delta(B/C) = \varepsilon \cdot (B/C) \quad (26)$$

The results for both of these cases are obvious from inspection of the benefit-cost expression. For the present purpose, Case 2 is more convenient and should suffice as an indicator of the benefit-cost uncertainty resulting from price uncertainty. For this case,

$$\frac{\Delta(B/C)}{(B/C)} = \frac{\Delta p}{p} \quad (27)$$

In estimating electricity price uncertainty over the next 5 to 15 years, several issues must be considered. How will regulatory pressures associated with combustion emissions and other environmental preservation issues affect generation? How will environmental issues affect transmission, and how will long-term costs associated with nuclear plants (i.e., decommissioning) actually be paid? What new technologies will enter the generation market? How will deregulation proceed and what will be its effects on price? It is difficult to attach numbers to any of these questions. Recent experience may be a useful guide—since 1980, there has been a 20% reduction in the real cost of electricity in the U.S. (Cavanaugh 1997). One cannot help but wonder what the predictions for the future price of electricity were in 1980—they were probably for increases in the real price, and, if so, the real uncertainty was actually larger than 20%. There is no apparent evidence that our ability to anticipate technological advance and/or regulatory evolution is better now than in 1980. It seems reasonable, therefore, to assume a price uncertainty of at least  $\pm 10\%$  for even relatively short-term analyses (i.e., five years). With this assumption, the fractional uncertainty in benefit-cost ratio due to uncertainty in electricity price is about  $\pm 10\%$ . One expects that for longer time horizons (i.e., > five years), considerably larger uncertainties in price would be appropriate.

### Discount Rate Uncertainty

The fractional uncertainty in the benefit-cost ratio can be written:

$$\frac{\Delta(B/C)}{(B/C)} = \Delta r \cdot G_{N,m,r} \quad (28)$$

where the function  $G_{N,m,r}$  is defined as

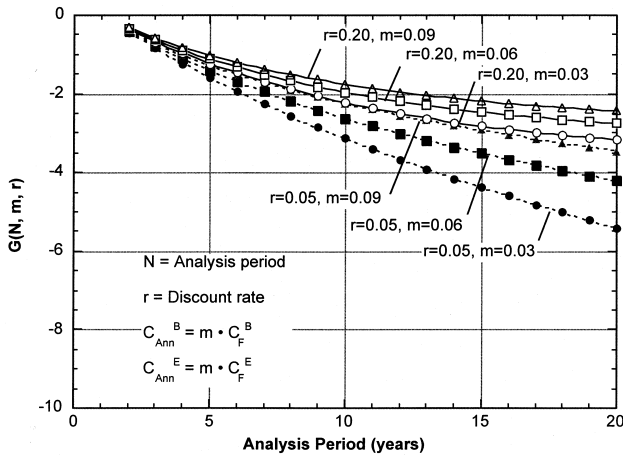
$$G_{N,m,r} = \frac{1}{1+m \cdot (1+D_{N-1,r})} \cdot \left( \frac{1}{1+r} + \frac{1}{D_{N,r}} \cdot \frac{\partial D_{N,r}}{\partial r} \right) \quad (29)$$

As before, the dimensionless parameter  $m$  is defined as the fraction of the technology first cost that is required for annual operating cost, which we assume to be the same for the base case and energy-efficient system options. Figure 3 shows numerical evaluation for parameter  $G_{N,m,r}$  as a function of  $N$  for values for  $m$  and  $r$  that span the likely range for these variables. For intermediate periods of analysis (i.e., 10 to 15 years) and discount rates between 5% and 20%,  $G_{N,m,r}$  ranges from about  $-4$  to  $-2$ .

Therefore,

$$2 \cdot \Delta r \leq \frac{\Delta(B/C)}{(B/C)} \leq 4 \cdot \Delta r \quad (30)$$

At a point in time, most owners will be able to define a discount rate for use in their financial analysis. In general, the discount rate will not be a concrete number and it will change over time in response to general economic conditions and to specific business conditions. Over a period of five years or longer, the discount rate uncertainty must be on the order of  $\pm r/2$ . If this is true, then the fractional uncertainty in the benefit cost ratio will range from  $\pm r$  to  $\pm 2r$ , depending on the owner's investment time horizon and the discount rate itself. Decision makers with large discount rates and short time horizons define the lower limit, while those with small discount rates and longer time horizons define the upper limit. If we take a 10% discount rate as typical, then the benefit-cost ratio uncertainty will be between  $\pm 10\%$  and  $\pm 20\%$ . For convenience, we will use  $\pm 15\%$  as the estimated uncertainty in the benefit-cost ratio due to uncertainty in the discount rate.



**Figure 3** Numerical evaluation for parameter  $G_{N,m,r}$  as a function of  $N$  for values for  $m$  and  $r$  that span the likely range for these variables.

## Analysis Period Uncertainty

The fractional uncertainty in the benefit-cost ratio is

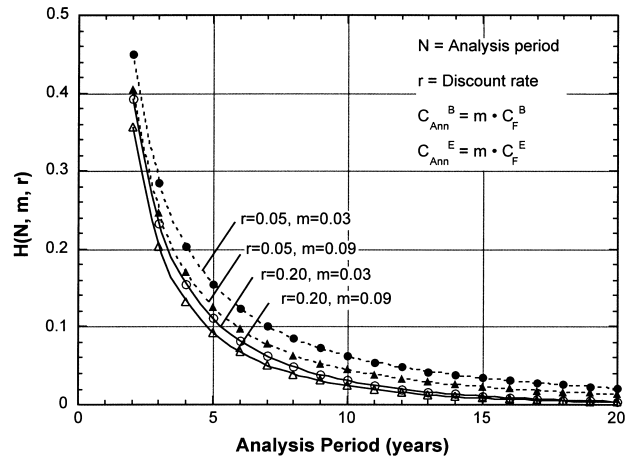
$$\frac{\Delta(B/C)}{(B/C)} = \Delta N \cdot H_{N,m,r} \quad (31)$$

where the function  $H_{N,m,r}$  is defined as

$$H_{N,m,r} = \frac{1}{1+m \cdot (1+D_{N,r})} \cdot \frac{1}{D_{N,r}} \cdot \frac{\partial D_{N,r}}{\partial r} \quad (32)$$

Again, the dimensionless parameter  $m$  is the fraction of the first cost required for annual non-energy operation and maintenance, and we assume the same value of  $m$  describes operating costs for both the base case and efficient equipment options

Figure 4 shows numerical evaluations for the parameter  $H_{N,m,r}$  as a function of  $N$  for values for  $m$  and  $r$  that span the likely range for these variables. For intermediate analysis periods (i.e., 10 to 15 years) and discount rates between about 5% and 20%,  $H_{N,m,r}$  will be between about 0.01 and 0.06. We estimate that  $\Delta N \approx \pm (N)/3$ . If we take 10 years as a typical investment time horizon, then the fractional uncertainty in the benefit-cost ratio will be in the range from  $\pm 3\%$  to  $\pm 20\%$ . Decision makers with large discount rates and longer time horizons define the lower bound, while the upper bound is defined by those with small discount rates and short time horizons. (Note from Figures 3 and 4 that the time horizons for the decision-making group associated with the upper and lower limits for discount rate uncertainty are the opposite of the time horizons for the groups that define the upper and lower limits for analysis period uncertainty. Both the group with long time horizons and the one with short time horizons will see substantial uncertainty in benefit-cost ratio associated with one of the two parameters.) For convenience, we will use  $\pm 10\%$  as the



**Figure 4** Numerical evaluations for the parameter  $H_{N,m,r}$  as a function of  $N$  for values for  $m$  and  $r$  that span the likely range for these variables.

estimated uncertainty in the benefit-cost ratio due to uncertainty in the period of analysis.

### Equipment Performance Uncertainty

The uncertainty considered in this section is the difference in equipment performance between design assumptions and actual operation. This will include deviations in chiller, cooling tower, and condenser water pump performance characteristics and system level effects. The uncertainties will include both deviations from manufacturers' published performance characteristics for new equipment and degradation of performance over the time period of the analysis. Design assumptions typically will be biased toward performance characteristics that are better than what is typically realized in practice; the bias is apt to be increased by expected performance degradation over time. Arriving at a defensible quantification of the extent of these uncertainties requires considerable guesswork.

In this analysis, we consider uncertainty in equipment performance for both the base case and efficient technology options. As with first cost and annual non-energy operating cost, we are dealing with the incremental performance effects. We define:

$$F_{L,i} = f_{B,L,i} - f_{E,L,i} \quad (33)$$

We assume  $F_{L,i}$  is a continuous function.<sup>5</sup> Proceeding as with first cost and annual non-energy operating cost, the uncertainty in the incremental performance,  $\Delta F_{L,i}$ , is expressed in terms of the uncertainty in the absolute performance of the base case and efficient equipment options. This yields

$$\frac{\Delta F_{L,i}}{F_{L,i}} \approx 1.4 \cdot \frac{\Delta f}{f_{B,L,i} - f_{E,L,i}} \quad (34)$$

where  $\Delta f$  is the uncertainty in the absolute performance of either the base case or efficient equipment option. If we assume that the efficient technology option provides a 20% performance improvement over the base case, then

$$\frac{\Delta F_{L,i}}{F_{L,i}} \approx \pm 7 \cdot \frac{\Delta f}{f} \quad (35)$$

The fractional uncertainty in the benefit-cost ratio becomes

$$\frac{\Delta(B/C)}{(B/C)} = \pm 7 \cdot \frac{\Delta f}{f} \quad (36)$$

In a situation where the performance advantage of the efficient equipment option over the base case is smaller than that assumed above (i.e., 10% instead of 20%), then the fractional uncertainty in the benefit-cost ratio will be larger than

<sup>5</sup> In some cases,  $F_{L,i}$  will be only piecewise continuous. For example, this will be true in a case where the base case and efficient equipment configurations include a different number of chillers and/or equipment staging is different.

indicated above (i.e., 100% larger!). Conversely, if the performance advantage of the efficient equipment option over the base case is larger (i.e., 30% instead of 20%), then the fractional uncertainty in the benefit-cost ratio will be smaller than indicated above (i.e., only 30% smaller).

The dominant component in a chilled water system from the kW/ton performance perspective is the chiller, which accounts for perhaps 70% of the total energy use of the system. We estimate the uncertainty in chiller performance as  $\pm 5\%$ , which assumes use of very effective design tools. For reference, ARI ratings for chillers require variations of 5% or less in measured performance at the rating conditions for otherwise identical machines. These values clearly would be optimistic when you consider performance differences between operation under test conditions and field applications and account for the integrated performance across all of the load, condenser, and supply water temperature conditions that will be experienced in application. Because they are a relatively small fraction of the total energy use of the chilled water system, we assume that the condenser water loop and cooling tower will add only slightly to the uncertainty in the overall system performance. We estimate the uncertainty in new construction to be  $\pm 6\%$ ; admittedly, this is a very optimistic estimate of a designer's ability to predict chilled water system performance.

Given this assumption, we estimate the uncertainty in the benefit-cost ratio due to uncertainty in system performance to be a minimum of  $\pm 35\%$ . This estimate treats the uncertainty as if it is random (i.e., normally distributed). If, instead, we assume that the error is systematic and has a rectangular probability distribution, then for the same assumptions, the benefit-cost uncertainty will increase to about  $\pm 50\%$ .

### Cooling Load Uncertainty

The sensitivity of the benefit-cost ratio to cooling load can be written as

$$\begin{aligned} \frac{\partial(B/C)}{\partial L} = & \frac{\sum_{i=1}^{8760} [(f_{B,L,i} - f_{E,L,i}) \cdot P_i] \cdot D_{N,r}}{\frac{C_F}{1+r} + C_{Ann} \cdot D_{N,r}} \\ & + \frac{\sum_{i=1}^{8760} [L_i \cdot \frac{\partial(f_{B,L,i} - f_{E,L,i})}{\partial L} \cdot P_i] \cdot D_{N,r}}{\frac{C_F}{1+r} + C_{Ann} \cdot D_{N,r}} \end{aligned} \quad (37)$$

In a simple case where the performance curves for the base case and efficient options (kW/ton vs. tons for the portion of the chilled water system being designed) are essentially parallel, then  $(f_{B,L,i} - f_{E,L,i})$  is constant, and the second term is 0. This assumption will be valid in a limited number of situations. In particular, it may hold in some cases where a BMS is being installed or where an existing control system is being enhanced. In the more general case, the curves will not be

parallel. For example, if one were to compare a design where there are multiple, staged chillers to one with a single larger chiller, then, for a typical load, the two systems will be in very different regions of the typical kW/ton curve. We have found no general, simple expression for the uncertainty in the benefit-cost ratio under these conditions. It is not clear that it is possible to do so without selecting specific system performance curves. For this reason, we will not include the uncertainty in the benefit-cost ratio due to load uncertainty in the summary below, and we will, therefore, be underestimating the actual uncertainty. Instead, we will use the uncertainties in the benefit-cost ratio due to the other parameters to define limits on the desired accuracy of the load data used in chilled water plant design.

## SUMMARY

In this study, uncertainty in the benefit-cost ratio and in simple payback have been estimated. The uncertainty is due to uncertainty in the parameters that enter the calculation. Table 1 summarizes the results for the six independent variables in the benefit-cost ratio for which estimates of uncertainty were possible. Calculations similar to those described in the previous section were also performed using the simple payback metric; these results are also included in the table.

As discussed earlier, uncertainties in the benefit-cost ratio and simple payback due to uncertainty in the cooling load profile could not be estimated without making specific assumptions about the performance of the chilled water system as a function of load. Adding the uncertainties for all other independent variables in quadrature results in total uncertainty of 54% and 59% for the benefit-cost ratio and simple payback, respectively. Because we have not included uncertainties associated with the cooling load, these total uncertainty estimates are conservative.

The key sources of risk in selection of an efficient chilled water system are equipment performance, first cost, and non-energy operating cost. This is not surprising since these parameters enter the calculation as incremental quantities—as differences between two values that are typically large relative

to the difference. Modest uncertainties in either of the values will result in appreciable uncertainty in the difference.

## CONCLUSIONS

Improved load and equipment performance data will allow designers to identify better chilled water system configurations and to identify better equipment, controls, and operating strategies; the tools will allow design of more energy-efficient systems. However, there is appreciable uncertainty in economic metrics commonly suggested and used. The uncertainty appears to be in the area of 50% even when using somewhat conservative estimates of component uncertainties. That is, even when the nominal benefit-cost ratio for the improved system is appreciably greater than 1, the decision maker is still at substantial risk that the additional investment will not meet the criteria.

In the analysis, we assumed an equipment performance uncertainty of  $\pm 5\%$ ; in spite of the fact that this is believed to be a far lower uncertainty than we can achieve with existing tools, this still translates to a large source of uncertainty in the decision metrics. Using engineering tools and data available today, the equipment performance uncertainty is probably greater than  $\pm 20\%$ , implying benefit-cost uncertainties on the order of 100%. We do need vastly improved equipment performance data and tools. One question that does remain is how far the tools and data issues can usefully be pushed before equipment dynamics must be considered. It is probable that this is not yet an issue.

From an engineering perspective, it is always tempting to tighten the engineering screws so that we have more accurate load data so that we have a better estimate of the energy cost savings. However, energy cost saving by itself is not an accepted decision-making metric, so the benefit of this approach is questionable. There is a reasonably well defined limit beyond which it is simply not useful to try to reduce the uncertainty in the load data that are used in developing a particular design. It appears that the inherent uncertainty limit for the benefit-cost ratio due to uncertainty in load data is

**TABLE 1**  
**Uncertainty Summary for New Construction**

Input Parameter		Benefit-Cost Ratio Uncertainty	Simple Payback Uncertainty
Identity	Uncertainty		
Incremental first cost	$\pm 5\%$	$\pm 25\%$	$\pm 35\%$
Incremental annual non-energy operating cost	$\pm 5\%$	$\pm 15\%$	$\pm 10\%$
Discount rate	$\pm r/2$	$\pm 15\%$	NA
Analysis period	$\pm N/3$ years	$\pm 10\%$	NA
Electricity price	$\pm 10\%$	$\pm 10\%$	$\pm 10\%$
Chilled water system equipment performance	$\pm 6\%$	$\pm 40\%$	$\pm 60\%$
Cooling load	TBD	TBD	TBD

about  $\pm 30\%$ . What does this mean? In a particular design, there is little or no benefit to the decision maker in using load profile data or load distribution data that are more accurate than is required to generate a benefits estimate within  $\pm 30\%$ .

Uncertainty in cost parameters can perhaps be reduced, but there is not much reason to be optimistic that this will actually happen in the foreseeable future. While it is quite likely that greater accuracy could be achieved through more comprehensive construction analysis and planning, it is not clear how far one can beneficially go in this direction. That is, there is a point where the incremental cost of decreasing risk exceeds the incremental benefit of reduced exposure. The fact that owners are willing to live with whatever is presently the typical level of contingency costs in construction projects suggests that this point may already have been reached.

The uncertainties in the economic metrics are the result of uncertainties in several of the input parameters, and at least some of these parameters are not controllable. That is, for uncontrollable parameters, such as discount rate and analysis period, we cannot beat down the uncertainty by improving any of our measurements or analytical techniques—these uncertainties must be lived with. Uncertainties in other parameters (e.g., equipment performance characteristics) can be controlled to some extent.

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